Chaotic dynamics of cold atoms in far-off-resonant donut beam

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Abstract

We describe the classical two dimensinal nonlinear dynamics of cold atoms in far-off-resonant donut beams. We show that there chaotic dynamics exists for charge greater than unity , when the intensity of the beam is periodically modulated. The two dimensional distributions of atoms in (x, y) plane for charge two are simulated. We show that the atoms will acumulate on several ring regions when the system enters to regime of global chaos.

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I. INTRODUCTION

The Lagurre-Gaussian beam carries orbital angular momentum associated with helical surface of constant phase has recently been the subject of considerable theoretical and experimental study[1-7]. Many of these studies concern the transfer of orbital angular momentum of the beam to small particals even atoms [1]. With the developement of laser cooling and trapping of neutral atoms, various schemes to slow the atomic motion and increase the atomic intensity have been proposed and some demonstarted. In a far-off-resonant laser beam it is possible to observe the spatial variation of the optical dipole force on slow atoms. In this case the atom experiences an effective mechanical potential propotional to the intensity of the beam. Recently the Gaussian-Laguerre modes were used as as optical potential to trap atoms [6].

The degree of nonlinearity of the potential is characterised by the 'charge' of the donut. The charge is an integer characterising the phase singularity of the beam on axis and which also determines the variation of the intensity off axia in the transverse plane. In the case of unit charge the intensity close to the axis varies quadratically with the radical distance in the transverse plane. For a charge greated than inity the variation of intensity is a quartic, or higher, power of the radical distance and thus the resulting motion near the beam axis is nonlinear. In this case the frequancy of bound oscillatory motion near the axis depends on the energy of the atom. As the energy increases, the frequency decreases, eventually falling to zero quite quickly as the unstable fixed point at the potential maximum is reached. It is well known that if such a nonlinear system is driven by a periodic modulation of the potential, chaotic dynamics can result [11]. In this paper we will consider the motion of cold atoms in intensity modulated Gaussian-Lagurre (donut) beam and discuss the classical two dimensional chaotic dynamics for charge $l = 2$.

II. LAGUERRE-GUASSIAN BEAM AND DONUT BEAM

The expression of a linearily poralized L-G beam in cylindral coordinator(r, ϕ, z) is [3]

$$\begin{aligned}\mathbf{E}(\mathbf{r}) = \mathbf{e}_x \frac{C}{1+z^2/{Z_R}^2}(\frac{r\sqrt{2}}{w(z)})^{|l|} {L_q}^{|l|}(\frac{2r^2}{w^2(z)}) \times \\ \exp(-r^2/w^2(z)) \exp[\frac{ikr^2 z}{2(z^2+z_R^2)}] \exp[il\phi] \times \\ \exp[i(2q+l+1)tan^{-1}(z/z_R)] \exp(ikz),\end{aligned} \tag{1}$$

where $z_R = \frac{\pi {w_0}^2}{\lambda}$, $w^2(z) = \frac{\pi(z^2+z_R^2)}{\lambda - R}$; $l = n - m$ and $q = min(n, m)$ for $E^{LG}_{m,n}$ mode.

We only concentrate on the donut beam, because it is the easiest and the most applicable way to form the optical trap, in which case it has $q = 0$ and therefore $L_0^{|l|}() = 0$. In reality (1) can be greatly simplified because in general $z \ll z_R$ and $w(z) \simeq w_0$.

The new expression of electric field for donut beam is

$$\mathbf{E}(\mathbf{r}) = \mathbf{e}_x C(\frac{r\sqrt{2}}{w_0})^{|l|} \exp(-r^2/w_0^2) \exp[il\phi] \exp(ikz), \tag{2}$$

where l is the topological charge of singularity for donut beam and the sign can be positive or negative. Each photon in the donut beam carrys $L_z = l\hbar$ orbital angular momentum for linearly polarized donut beam. The intensity distribution of the $LG_{q=0}$ beam is given by [6]

$$I(\mathbf{r}) = W \frac{2^{|l|+1} r^{2|l|}}{\pi |l|! w_0^{2(|l|+1)}} \exp[-2r^2/w_0^2], \tag{3}$$

where W is the power of laser.

III. CHAOTIC DYNAMICS OF COLD ATOMS IN FAR-OFF-RESONANT DONUT BEAM

For cold atoms in far-off-resonant donut beam, the spontaneous emission can be ignored. The effective optical potential for two-level atom [8] has the form

$$U(\mathbf{r}) = \frac{\hbar\Delta}{2} ln(1+p), \tag{4}$$

where Δ is the detuning and $p = \frac{\Omega^2/2}{\Delta^2+\Gamma^2/4}$ is a saturation parameter, where Ω is Rabi frequency, for for-off-resonant donut beam $p \ll 1$ and thus

$$U(\mathbf{r}) = \frac{\hbar\Omega(\mathbf{r})^2}{4\Delta}. \tag{5}$$

Taking into account the special variation of the Rabi frequency, the classical Hamiltonian in (x, y) plane for the system is

$$H_0 = \frac{p_x^2 + p_y^2}{2M} + K(l)(x^2 + y^2)^l \exp(-2\frac{x^2 + y^2}{w^2}), \tag{6}$$

where $K(l) = \frac{\hbar\Omega_0^2}{2\Delta}\frac{2^l}{l! w_0{}^{2l}}$ and $\Omega_0 = \Gamma\sqrt{\frac{W}{2\pi w_0{}^2 I_s}}$, I_s is the saturation intensity, for Rb atom $I_s = 2mW/cm^2$.

When the Rabi frequency is modulated and Ω_0 is substitued by $\Omega_0\sqrt{1 + \epsilon\cos(\omega t)}$,, the dynamics of the atom is chaotic . For the different charge l, the potential is different. The larger the value of l, the wider the bottom of the potental. For $l = 1$, at the bottom of this potental the motion of atom is harmonic and no chaotic dynamics for periodic modulated potential can arise. Only if the atom has a larger kinetic energy so that the nonlinear parts of the potential are expected, does the dynamics became chaotic. For $l > 1$, the dynamics is nonlinear over the whole potential range and because the bottom is wider, it is easier to control the amplitude of modulation.

For simplicity we discuss $l = 2$ case and the practical parameters for our numerical examples are that for Rb atoms, the linewidth $\Gamma/2\pi = 6MHz$, mass $M = 85m_p$, the beam waist of laser $w_0 = 140\mu m$, laser power $W = 600mW/cm^2$ and the detuning $\Delta/2\pi = 6GHz$. We define dimensionless parameters $(x, y) = (\tilde{x}, \tilde{y}) = (x/w, y/w)$, $(\tilde{p_x}, \tilde{p_y}) = (p_x/P_D, p_y/P_D)$, and $\tilde{H} = H/2E_D$ and $\tilde{t} = t/\frac{w}{MP_D}$, where $E_D = \frac{P_D^2}{2M}$ is the Doppler limit energy and P_D is the momentum respectively. Omitting the tildes and defining $\xi = \frac{\hbar\Omega^2}{2\Delta E_D}$, the Hamiltonian can be rewritten for charge $l = 2$ as

$$H(t) = \frac{p_x^2 + p_y^2}{2} + \xi(l)(x^2 + y^2)^2 e^{-2(x^2+y^2)}(1 + \epsilon\cos\omega t), \tag{7}$$

where $\xi \approx 0.887$. Using Hamiltons equations we find that the motion in the transverse plane without modulation ($\epsilon = 0$ is described by the equations,

$$\dot{p_x} = -4\xi x r^2(1-r^2)e^{-2r^2}, \tag{8}$$

$$\dot{p_y} = -4\xi y r^2(1-r^2)e^{-2r^2}, \tag{9}$$

$$\dot{x} = p_x, \tag{10}$$

$$\dot{y} = p_y, \tag{11}$$

where $r^2 = x^2 + y^2$. Clearly there are twofixed points, one stable fixed point on axis ($r = 0$), and one unstable fixed point at the intensity maximum (r=1).

The chioce of modulation frequency ω depends on the frequency of unperturbed periodic motion. For simplicity we assume $y = 0$ and $p_y = 0$, so the expression for H is simplified as one dimensional Hamiltanian. The motion period for unperturbed Hamiltanian H_0 is [9]

$$T = \oint \frac{dx}{\partial H_0/\partial p_x} = 2\int_{-x_M}^{x_M} \frac{dx}{\sqrt{2(H_0 - \xi x^2 \exp{(-2x^2)}}}, \tag{12}$$

where x_M is determined by $H_0 = \xi x_M^2 \exp(-2x_M^2)$. Therefore

$$\omega_0 = \frac{\pi}{\int_{-x_M}^{x_M} \{2[H_0 - \xi x_M^2 \exp{(-2x_M^2)}]\}^{-1/2} dx}. \tag{13}$$

The graph of ω versus H_0 and x_M versus H_0 can be seen in fig.2. We can select the modulation frequency ω to control the fix points in periodic optical potential. Here we set dimensionless parameter $\omega = 4.34$, which corresponds to $3.67KHz$.

We use the symplectic integrators [10], [11] to solve the equations of motion because the Hamiltonian evolution preserves the Poisson bracket relation $\{x(t), p_x(t)\} = 1$

We plot the stroboscopic potrait of the system at times $t = (2\pi/\omega)s$, where s is an integer referred to as the strobe number. From figure 3,4,5 we can see that with the increase of ϵ, the motion of atoms will appear chaos. But there are some stable regions. An initial wide spectral distribution of atoms will result in some atoms trapped in these stable regions.

Laser cooling and trapping techniques have the ability to cool the atom to very low velocities and trap them in very small region in momentum. Therefore the appropriate description of atomic dynamics is to use probability distribution on phase space (x, y, p_x, p_y).

We define a classical state to be a probability measure on phase space of the form $Q(x,y,p_x,p_y)dxdydp_xdp_y$, the density of probability satisfies the Liouville equation

$$\frac{\partial Q}{\partial t} = \{H, Q\}_{q_i,p_i}, \tag{14}$$

where$\{,\}_{q_i,p_i}$ is the Poisson bracket. The equation can be solved by the method of characteristics. To simulate the experiment, we assume initially atoms are umiformly distributed on $|x| < C$ and $|y| < C$ region, where C is a constant choosen to ensure the major fixed points are included. The momentum distributions for p_x and p_y are Gaussian distributions. Therefore

$$Q_0(x,y,p_x,p_y) = Q_0(x)Q_0(y)Q_0(p_x)Q_0(p_y), \tag{15}$$

where

$$Q_0(p_i) = \frac{1}{2\pi\sigma_{p_i}} \exp\left[-(p_x - p_x(0))^2/2\sigma_{p_i}\right]. \tag{16}$$

The variances of p_x and p_y are related to the temperature T_i

$$\sigma_{p_i} = k_B T_i / P_D^2. \tag{17}$$

The two dimensional symplectic integrators [10] are used to keep the Poisson bracket relations during computation

$$\{q_i, p_j\} = \delta_{ij} \tag{18}$$

Fig. 6 shows for optical potential when it is no modulation, atoms will accumulate around the fix point$x = y = 0$ (fig.3). when the modulation is added the atoms will diffuse but will accumulate around several rings. With the increase of modulation amplitude, more atoms accumulate around rings and less atoms around $x = y = 0$ (fig.7,8). The variances σ_{p_x} and σ_{p_y} are taken as 0.05 in computation, which corresponds to temperature T_x and T_y approximately recoil temperature. If the variance is narrower rings become clearer.

IV. CONCLUSION AND DISCUSSION

In summary, we have shown that it has chaotic dynamics for mudulated far-off-detuning donut beam. For atomic momenta p_x, p_y which have Gaussian distributions, atoms will be trapped in rings when the optical potential is modulated. If at some moment the optical potential is withdrawn, atoms will expand freely and will keep the shape of rings becasue the momentua is symmetric in (x, y). Therefore the two dimensional atomic distribution in (x, y) can be detected using TOF technique.

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FIGURES

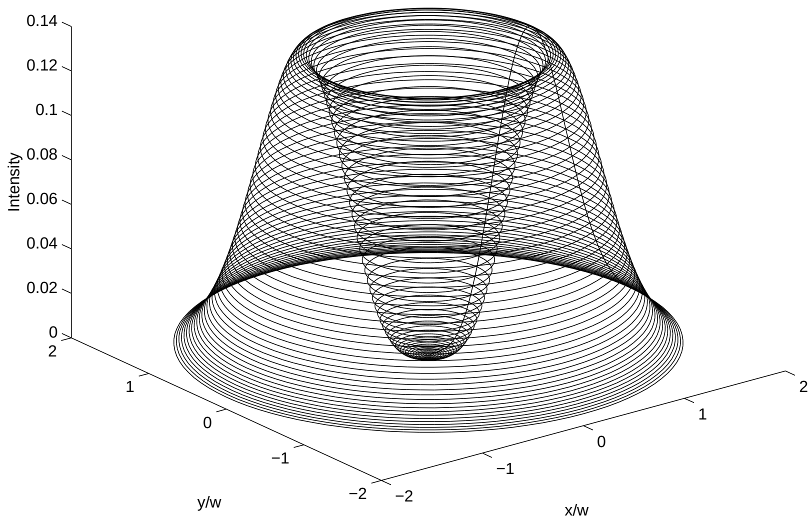

FIG. 1. *The relative intensity of donut beam for charge $l = 2$, where w is the beam waist.*

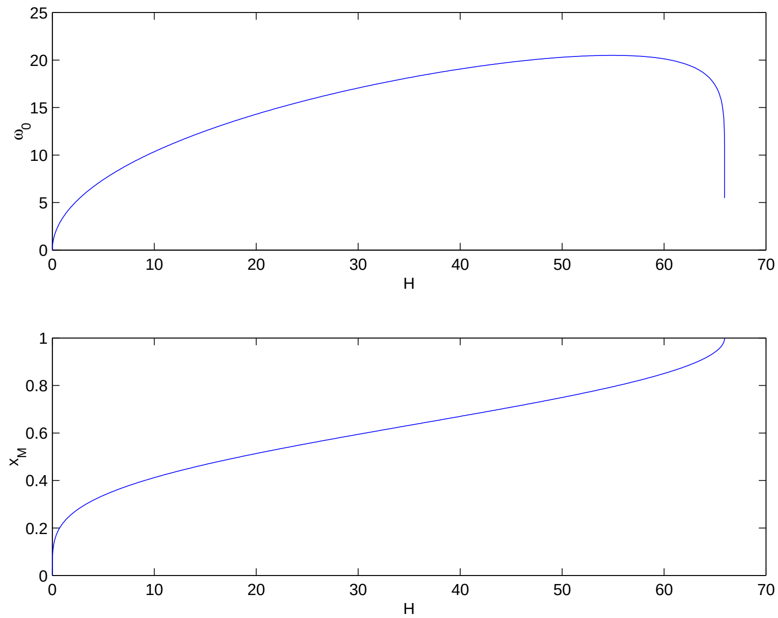

FIG. 2. *The frequency of motion* $\omega_0 \sim H$ *and* $x_M \sim H$.

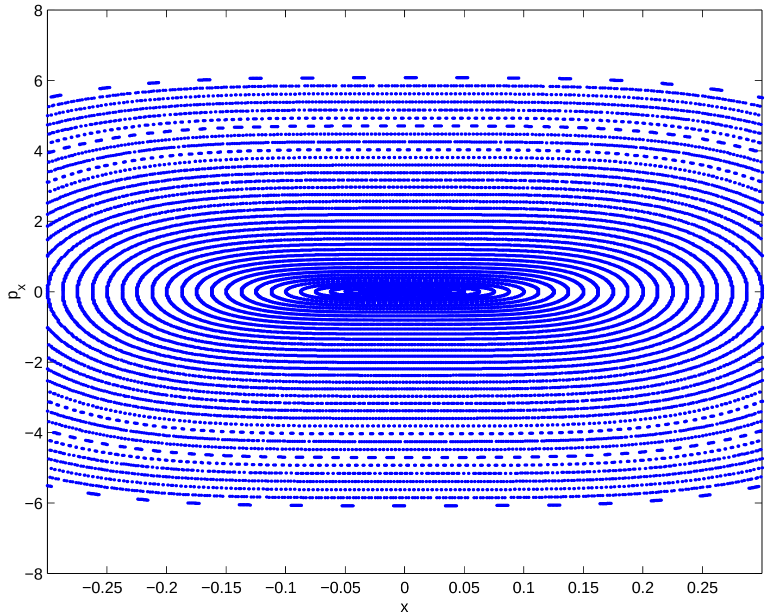

FIG. 3. *Stroboscopic potrait of the system with* $\epsilon = 0$, $p_x(0) = 0$,$p_y =$ *and* $y = 0$. *The maximun strobe number is* 500.

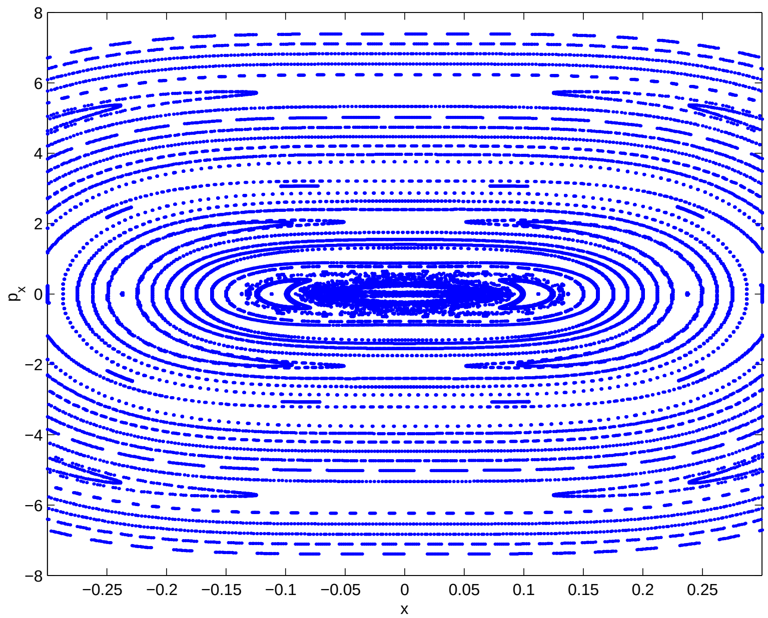

FIG. 4. *Stroboscopic potrait of the system with* $\epsilon = 0.5$, $p_x(0) = 0$,$p_y = 1$ *and* $y = 0$.*The maximun strobe number is* 500.

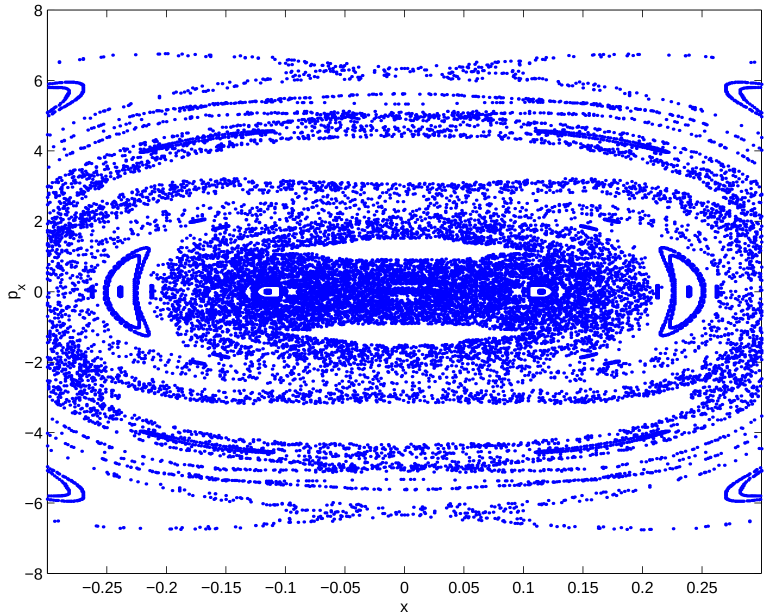

FIG. 5. *Stroboscopic potrait of the system with* $\epsilon = 0.7$, $p_x(0) = 0$,$p_y = 1$ *and* $y = 0$.*The maximun strobe number is* 500.

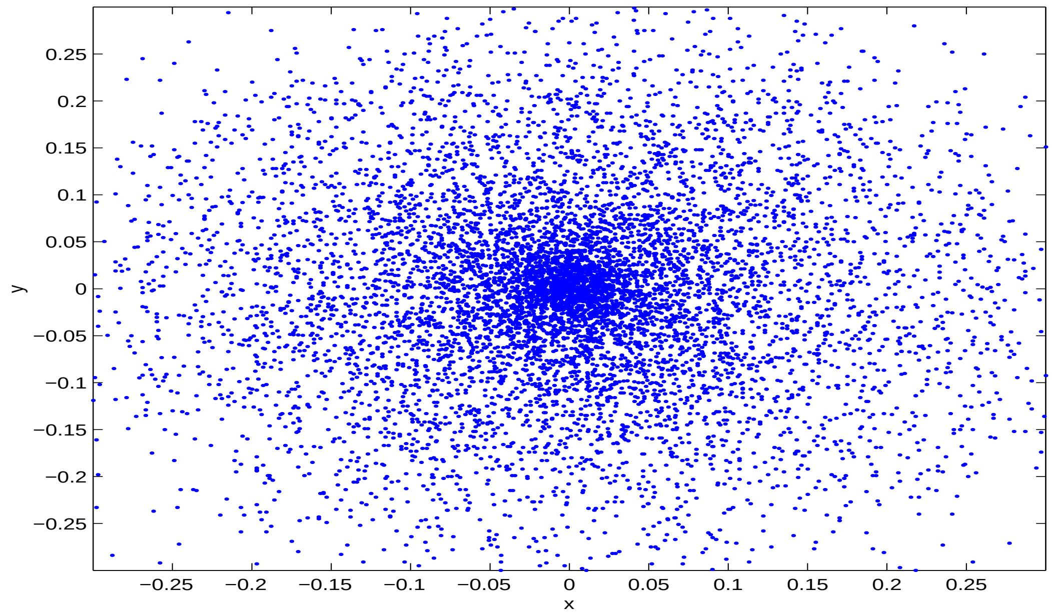

FIG. 6. *The atomic distribution in* (x, y) *plane at the strobe number 50 for* $\epsilon = 0$*. The 10000 points were taken in phase space. The atoms were initially distributed on* $|x| \leq 0.3$ *and* $|y| \leq 0.3$ *region. The momenta of* p_x*,* p_y *are Guassian distributions and* $\sigma_{p_x} = \sigma_{p_y} = 0.05$*.*

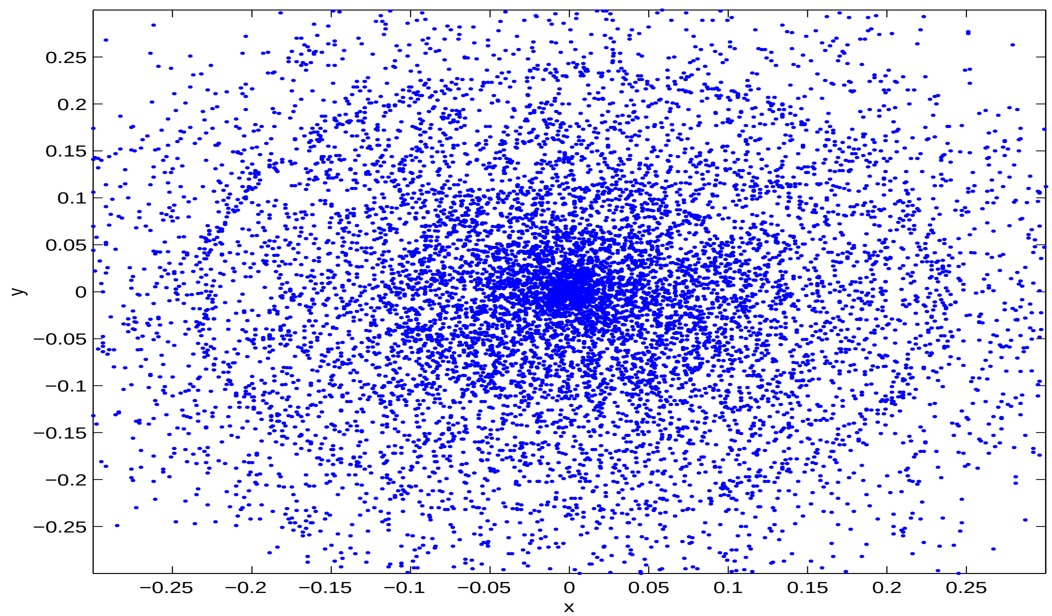

FIG. 7. *The atomic distribution in (x, y) plane at the strobe number 50 for $\epsilon = 0.5$. The 10000 points were taken in phase space. The atoms were initially distributed on $|x| \leq 0.3$ and $|y| \leq 0.3$ region. The momenta of p_x, p_y are Guassian distributions and $\sigma_{p_x} = \sigma_{p_y} = 0.05$.*

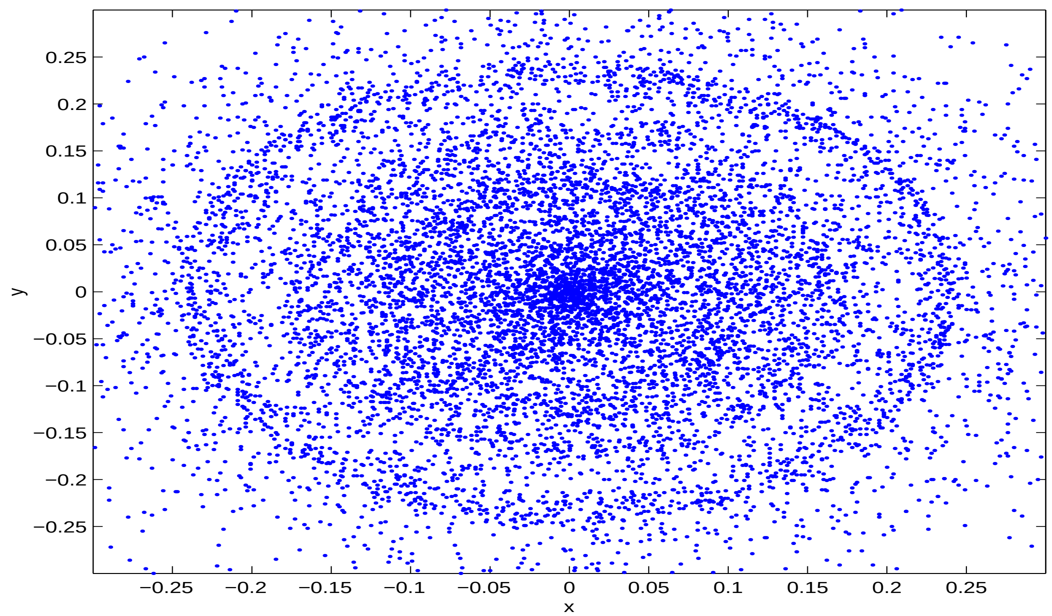

FIG. 8. *The atomic distribution in* (x, y) *plane at the strobe number 50 for* $\epsilon = 0.7$*. The 10000 points were taken in phase space. The atoms were initially distributed on* $|x| \leq 0.3$ *and* $|y| \leq 0.3$ *region. The momenta of* p_x*,* p_y *are Guassian distributions and* $\sigma_{p_x} = \sigma_{p_y} = 0.05$*.*